(4.) QN. → Test the Convergency of the series

$$\frac{2}{1}+\frac{7}{15}+\frac{12}{53}+\frac{17}{127}+\frac{22}{249}+\cdots+\frac{5n-3}{2n^3-1}+\cdots$$

Ans. → Here, $u_n=\dfrac{5n-3}{2n^3-1}$

$$=\frac{\cancel{n}\left(5-\frac{3}{n}\right)}{\cancel{n^3}\,n^2\left(2-\frac{1}{n^3}\right)}=\frac{\left(5-\frac{3}{n}\right)}{n^2\left(2-\frac{1}{n^3}\right)}$$

Let us consider an Auxiliary series, whose n^{th} term is $v_n=\dfrac{1}{n^2}$

$$\frac{u_n}{v_n}=\frac{\dfrac{\left(5-\frac{3}{n}\right)}{n^2\left(2-\frac{1}{n^3}\right)}}{\dfrac{1}{n^2}}=\frac{\left(5-\frac{3}{n}\right)}{\cancel{n^2}\left(2-\frac{1}{n^3}\right)}\times\frac{\cancel{n^2}}{1}$$

$$\lim_{n\to\infty}\frac{u_n}{v_n}=\lim_{n\to\infty}\frac{\left(5-\frac{3}{n}\right)}{\left(2-\frac{1}{n^3}\right)}=\frac{\left(5-\frac{3}{\infty}\right)}{\left(2-\frac{1}{\infty^3}\right)}=\frac{\{5-0\}}{\{2-0\}}=\frac{5}{2}$$

$=\frac{5}{2}$ which is finite and non-zero

∴ from Comparison test, Σu_n and Σv_n will be converget and divergent simultaneously.

But, $v_n=\dfrac{1}{n^2}$ which is converget

Hence, by Comparison Test,
Σu_n is also Convergent.

(3) QNo→ Test the Convergence of $\sum_{n=1}^{\infty} \frac{2n^2+1}{3n^3+5n^2+6}$

Ans.→ Here, $u_n = \frac{2n^2+1}{3n^3+5n^2+6}$

$= \frac{n^2(2+\frac{1}{n^2})}{n^3(3+\frac{5n^2}{n^3}+\frac{6}{n^3})}$

$= \frac{1}{n} \frac{(2+\frac{1}{n^2})}{(3+\frac{5}{n}+\frac{6}{n^3})}$

Let us Consider an Auxiliary series, whose n^{th} term is $v_n = \frac{1}{n}$

$\frac{u_n}{v_n} = \frac{\frac{(2+\frac{1}{n^2})}{n(3+\frac{5}{n}+\frac{6}{n^3})}}{\frac{1}{n}} = \frac{(2+\frac{1}{n^2})}{n(3+\frac{5}{n}+\frac{6}{n^3})} \times \frac{n}{1}$

$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{(2+\frac{1}{n^2})}{(3+\frac{5}{n}+\frac{6}{n^3})} = \frac{(2+\frac{1}{\infty^2})}{(3+\frac{5}{\infty}+\frac{6}{\infty^3})} = \frac{(2+0}{3+0+0} = \frac{2}{3}$

$= \frac{3}{2}$ which is finite and non zero

∴ from Comparison test Σu_n and Σv_n will be converget and divergent simultaneously.

But, $v_n = \frac{1}{n}$ which is divergent

Hence, by Comparison Test
Σu_n is also divergent.

(5) QNo→ Test the Convergence of the series.

$1 + \frac{3}{5} + \frac{5}{13} + \frac{7}{25} + \frac{9}{41} + \cdots + \frac{2n-1}{2n^2-2n+1} + \cdots \text{ to } \infty.$

Ans.→ Here, $u_n = \frac{2n-1}{2n^2-2n+1} = \frac{n(2-\frac{1}{n})}{n^2(2-\frac{2n}{n^2}+\frac{1}{n^2})}$

$$= \frac{\cancel{n}\left(2-\frac{1}{n}\right)}{n^{\cancel{2}}\left(2-\frac{2}{n}+\frac{1}{n^2}\right)}$$

Let us Consider an Auxilliary series, whose nth term is $v_n = \frac{1}{n}$

$$\frac{u_n}{v_n} = \frac{\frac{\left(2-\frac{1}{n}\right)}{n\left(2-\frac{2}{n}+\frac{1}{n^2}\right)}}{\frac{1}{n}} = \frac{\left(2-\frac{1}{n}\right)}{\cancel{n}\left(2-\frac{2}{n}+\frac{1}{n^2}\right)} \times \frac{\cancel{n}}{1}$$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{\left(2-\frac{1}{n}\right)}{\left(2-\frac{2}{n}+\frac{1}{n^2}\right)} = \frac{\left(2-\frac{1}{\infty}\right)}{\left(2-\frac{2}{\infty}+\frac{1}{\infty^2}\right)}$$

$$= \frac{2-0}{2-0+0} = \frac{\cancel{2}}{\cancel{2}} = 1$$

$= 1$ which is finite and non zero

$\therefore$ from Comparison test, Σu_n and Σv_n will be Convergent and divergent simultaneously.

But $v_n = \frac{1}{n}$ which is divergent.

Hence, by Comparison Test

Σu_n is also divergent.

P.U. 12 Q.No → Test whether $\frac{2}{1}+\frac{3}{4}+\frac{4}{9}+\frac{5}{16}+\frac{6}{25}+\text{------}$ is convergent or not?

Ans. → ~~nth~~ Let the nth term of the given series be denoted by u_n.

Here, $u_n = \frac{n+1}{n^2}$

$$= \frac{\cancel{n}\left(1+\frac{1}{n}\right)}{\cancel{n^2}\, n}$$

$$= \frac{\left(1+\frac{1}{n}\right)}{n}.$$

nth term of $2+3+4$ ---

$a = 2$

$d = 1$

$\therefore a+(n-1)d$

$= 2+(n-1)\cdot 1$

$= 2+n-1$

$= n+1$

1^2; 2^2, 3^2

Let, n^2

Let us consider an Auxiliary series, whose n^{th} term is $v_n = \frac{1}{n}$

$$\frac{u_n}{v_n} = \frac{\frac{(1+\frac{1}{n})}{n}}{\frac{1}{n}} = \frac{(1+\frac{1}{n})}{\cancel{n}} \times \frac{\cancel{n}}{1}$$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right) = \left(1+\frac{1}{\infty}\right) = 1+0 = 1$$

$= 1$ which is finite and non zero

$\therefore$ from comparison test, Σu_n and Σv_n will be convergent and divergent simultaneously.

But $v_n = \frac{1}{n}$ which is divergent.

Hence, by comparison test

Σv_n is ~~and~~ also divergent.

7 QNo → Test the convergency of the series.

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots \text{ to } \infty.$$

Ans. → Let the n^{th} term of the given series by denoted by u_n

n^{th} term of $1, 3, 5 \cdots$

$a = 1$

$d = 2$

$\therefore a + (n-1)d$

$= 1 + (n-1)\cdot 2$

$= 1 + 2n - 2$

$= 2n - 1$

Let, $v_n = \frac{1}{n^2}$

Here, $u_n = \frac{1}{(2n-1)^2}$

$v_n = \frac{1}{n^2}$

$$\therefore \frac{u_n}{v_n} = \frac{1}{(2n-1)^2} \times \frac{n^2}{1} = \frac{n^2}{(2n-1)^2}$$

$$= \frac{\cancel{n^2}}{\cancel{n^2}(2-\frac{1}{n})^2} = \frac{1}{(2-\frac{1}{n})^2}$$

Let us consider an Auxiliary series, ~~whose nth term~~

is $\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{(2-\frac{1}{n})^2} = \frac{1}{(2-\frac{1}{\infty})^2} = \frac{1}{(2-0)^2}$

$= \frac{1}{4} = \frac{1}{4}$ which is finite and non zero

$\therefore$ from comparison test, Σu_n and Σv_n will be convergent and divergent simultaneously.